We obtain the estimate of the maximum size of the domain of solution of (5) by analytic continuation of the function Y to the domain of the complex variable $\xi = \xi_1 + i\xi_2$, $\varphi = \varphi_1 + i\varphi_2$. Here (5) corresponds to the wave equation /8/ $\partial^2 Y/\partial\varphi_1^2 - \partial^2 Y/\partial\xi_2^2 = F(\xi)$. Its characteristics are determined by the expression $d\xi_2/d\varphi_1 = \pm 1$, which forms a family of parallel lines parallel to the bisectrices of the coordinate angles whose apices satisfy the relation $\xi_2 \pm \varphi_1 = \text{const}$. From the first boundary condition of (6), by equating ξ_2 to the magnitude of the segment of the known part of the boundary (Fig.1,2) and the characteristic passing through until it intersects the φ_1 axis, we obtain an approximate estimate of the maximum size of the unknown domain of solution of $\varphi_0^{(2)} = 0.396$, i.e. the difference between it and the result obtained earlier using the method of integral equations, does not exceed 3.5%. The comparison shows the possibility of using the method of characteristics to solve elliptical boundary value problems with an unknown boundary in the theory of thin shells.

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VIBRATION OF AN ELASTIC ROD WITH DRY FRICTION ON ITS SIDE SURFACE*

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Steady longitudinal oscillations in a semibounded elastic rod are studied, taking into account "dry" friction on its side surface. An approximate solution is constructed using the method of harmonic linearization /1/ which leads to a boundary value problem for a system of two non-linear equations. The latter can be reduced to the Cauchy problem by a change of variables. Results of numerical computations are given.

We consider the longitudinal oscillations of a weightless one-dimensional elastic rod of constant cross-section, taking into account dry friction on its side surface. Steady oscillations are discussed, unlike in /2/ where a problem with initial data was solved for the case when the end face of the rod was loaded according to special laws. We specify a harmonic perturbation of the deformation at one of its ends and assume the other end (removed to infinity) to be at rest, to obtain the system

$$\rho S \partial^2 u/\partial t^2 = ES \partial^2 u/\partial x^2 - q \operatorname{sign}(\partial u/\partial t) \tag{1}$$
$$x = 0,\ u = u_0 \cos \omega t;\ x \to \infty,\ u \to 0\ (u_0 \equiv \text{const} > 0)$$

where u, S denote the displacement and the area of transverse cross-section, ω is the

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frequency, ρ, E are the density and Young's modulus and q is the magnitude of the force of friction on the side surface per unit length of the rod.

We shall seek the approximate solution of problem (1) in the form

$$u = u_0 v(x) \cos[\omega t + \varphi(x)] \tag{2}$$

and

$$v \equiv 0, \quad 0 < x_* \leqslant x \tag{3}$$

(x_* is an unknown quantity) and we apply in the region $0 \leqslant x < x_*$ the method of harmonic linearization /1/ in which representation (2) is used. Then from (1)-(3) we obtain the following system of equations and boundary conditions:

$$v\varphi'' + 2v'\varphi' = 1/p, \quad v'' + [1 - (\varphi')^2] v = 0 \tag{4}$$

$$v(0) = 1, \; v(z_*) = 0, \; v'(z_*) = 0, \; \varphi(0) = 0 \tag{5}$$

$$z = \sqrt{\rho/E}\,\omega x, \quad z_* = \sqrt{\rho/E}\,\omega x_*, \quad p = {}^1/_4 \pi\rho s u_0 \omega^2/q$$

The new dimensionless coordinate z serves as the argument of the functions v, φ, and the prime denotes differentiation with respect to z. The third condition in (5) follows from (3) and from the continuity, when $x = x_*$, of the intensity of the force in the elastic body proportional to the derivative $\partial u/\partial x$. The missing boundary condition for (4) follows from the requirement that $\partial u/\partial x$ be bounded

$$|v'(z)| < \infty, \quad |v(z)\varphi'(z)| < \infty, \quad 0 \leqslant z \leqslant z_* \tag{6}$$

In deriving (4) we assumed that

$$v(z) > 0, \quad 0 \leqslant z < z_* \tag{7}$$

We will first give the solution of problem (4)-(7), asymptotically exact when $p \ll 1$

$$v = (1 - z/z_*)^2, \quad \varphi = \sqrt{2}\ln(1 - z/z_*), \quad z_* = (\sqrt{18p})^{1/2} \tag{8}$$

Thus, taking the third condition of (7) into account we find that the functions v and φ differ significantly from their approximate linear expressions in /3/.

Turning now to the case of arbitrary p, we can confirm that the first equation of (4) has an integral

$$-\varphi' = \frac{g}{(pv)^2} \quad \left(g = \text{const} - \int_0^z pv\,dt\right) \tag{9}$$

Substituting (9) into the second equation of (4), we obtain

$$g''' + g' - g^2/(g')^3 = 0 \tag{10}$$

In order to satisfy the boundary conditions (5) and (7), we shall require that

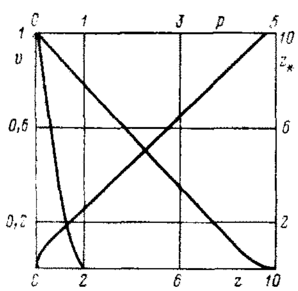
$$g(z_*) = 0, \; g'(z_*) = 0, \; g''(z_*) = 0, \; g'(0) = -p \tag{11}$$

$$g'(z) < 0, \; 0 \leqslant z < z_* \tag{12}$$

and conditions (6) hold in this case independently.

Our principal aim will be to establish the dependence of z_* on p. In order to determine this relation, we shall reverse the formulation (11), i.e. we shall assume that z_* is given and p is unkown, and replace the third condition in (11) by $g''(z_*) = w$ where w is an arbitrary positive constant. Then instead of (11) we obtain

$$g(z_*) = 0, \; g'(z_*) = 0, \; g''(z_*) = w, \quad w > 0 \tag{13}$$



The Cauchy problem (10), (12), (13) is convenient for numerical work using a computer. Here $g'(0)$ should obviously be taken as the parameter p. The passage to the limit $w \to 0$ returns us to conditions (11).

The computations were carried out using a library program with automatic selection of a step, by reducing the conditions (10) to a system of three first-order equations. Expanding $g(z)$ in a power series in the neighbourhood of $z = z_*$, we find that $g^2/(g')^3 \to 0$ as $z \to -z_*$, therefore the above fraction was made equal to zero in the numerical algorithm for $z = z_*$. When $p \ll 1$, the asymptotic and numerical values of $z_*(p)$ in (8) were practically the same.

The figure shows the relation $z_*(p)$ and $v(z)$. It is clear that the solution obtained also holds for a rod of finite length l and for the same values of p, as long as $x_* < l$.

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